Grade Level/Course: Algebra II/Trigonometry

Lesson/Unit Plan Name: What is a Radian?

Rationale/Lesson Abstract: Students will be able to define a radian. Students will be able to determine in which quadrant an angle (in radians) lies.

Timeframe: 1 period (55 minutes)

Common Core Standard(s):

F-TF.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Instructional Resources/Materials:
• A variety of circular objects (see photo below)

• Adding machine tape

• Rulers (with cm)
• Scissors
• Markers or colored pencils (nice to have, not necessary)
• Copies of student's guided notes
• Copies of exit tickets

Teacher's Guide

Group students so that you have an equal number of groups and circular objects. (Ex: If you brought in 6 different circular objects and you have 30 students in your class, make groups of 5 students so that you have 6 groups total.)

Each group should have:

- 1 circular object
- 1 pair of scissors
- 1 or more rulers (students can share a ruler in the group)
- markers or colored pencils

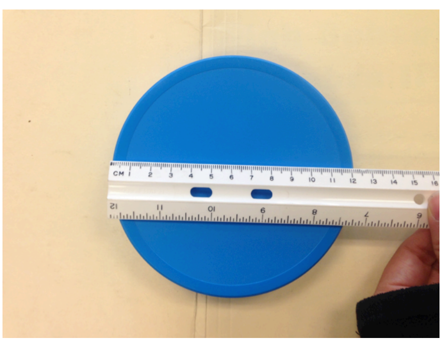
Have students follow along with their guided notes as you give direction and guide instruction.

Activity: Discovering what a radian is.

Each group should measure the diameter of their circular object and record it in their notes. The group has multiple rulers (or will share a ruler) so that at least 2 students can measure the diameter to reduce error.

Use the group agreed upon measure of diameter (average, or most accurate) to determine the radius of the circular object.

Example Student Work



Group's agreed upon measure.

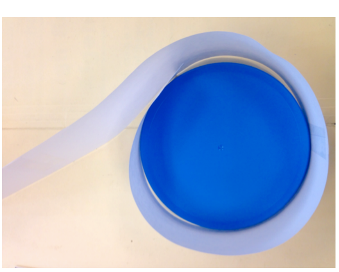
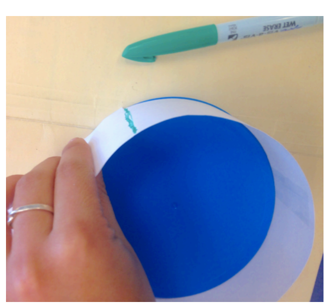
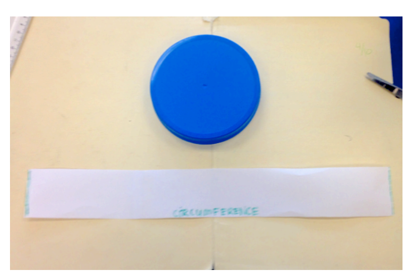
Measure the diameter of your circular object:

12.2 cm	12 cm	12.4 cm
Student 1's measure	**Student 2's measure**	**Student 3's measure**

Use the group's agreed upon measure of the diameter to find the radius. RADIUS (r) = 6.1 cm

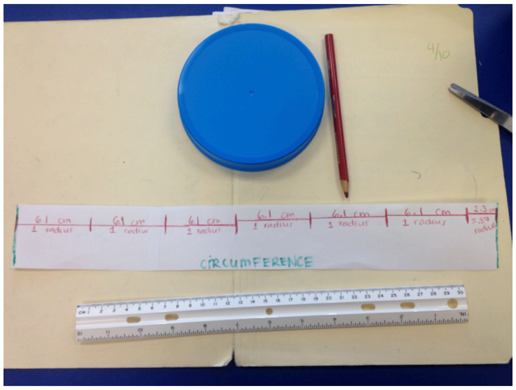
Measure the circumference of your circular object by rolling the strip of paper around the edge. Cut the strip of paper to the exact measure of the circumference.

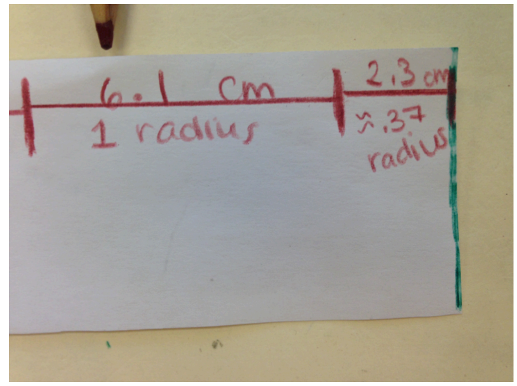
Example Student Work:

 →  → 

Measure and mark off radius measurements on your circumference paper. Include centimeter measures and determine how many radius measures fit into your circumference. If it's not a whole number, estimate what portion of a radius fits in at the end. Record these numbers into the first line of your data table.

Example Student Work:





Object	Radius length	# of times radius fits into circumference
Blue lid	6.1 cm	≈ 6.37 times

Once students are done taking these measurements, have groups share out (either whole class or share with each other) their observations and let students record the measurements of other objects to fill in the rest of their table with data.

Example Student Work:

Object	Radius length	# of times radius fits into circumference
Blue lid	6.1 cm	≈ 6.37 times
Green coffee mug	4.7 cm	≈ 6.2 times
Cookie tin	13.5 cm	≈ 6.5 times
Bottom of the globe	10.5 cm	≈ 6 times

Facilitate a student discussion of why each object, no matter its size, has a similar number of times that the radius fits into the circumference. Ask students in groups to discuss if that number has any significance to them. Maybe it's something they learned in Geometry? ** If students need more guidance, have them divide that number by π (approximately 3.14) and make an observation. Have them record their group discussion in their guided notes.

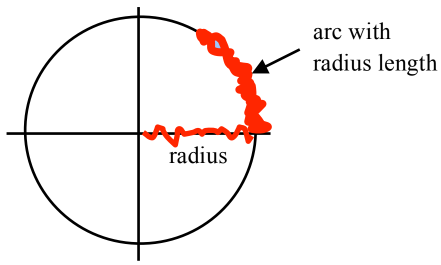
Example Student Work:

About how many radius lengths fit in your circumference measurement? About 6.3 or 6.4

Does this number remind you of anything you learned in Geometry? Yes it does!

If so, what? This number is close to 2 times 3.14 or 2π.

Have students estimate the length of the radius of the circle on their guided notes, then have them draw an arc (from 0º) that is approximately this radius length. Lead a whole class discussion (use strategies such as think-pair-share and small group discussions) about the three follow-up discussion questions.



What would the angle measure be for an angle to subtend this arc?

We don't know exactly, but it looks like about 60º.

Would the angle be different depending on the size of the circle?

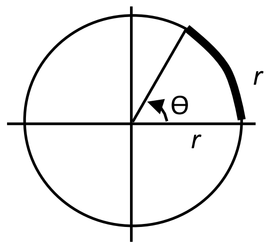
No.

Why or why not?

Because all circles are similar and the radius lengths would be proportional, therefore, the angle measure would remain the same.

Example Student Work:

An angle that measures __1 radian__ subtends an arc that is __1 radius__ long.



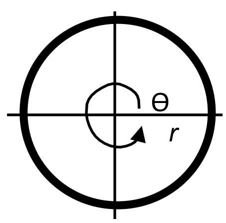
Θ = __1 radian__

Extend this discussion to the entire circle, so that we have radian measures for what we normally look at as degrees.

Example Student Work:

How many radians are in a full revolution?

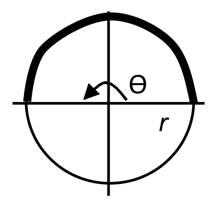
Θ = __2π radians__



How many radians are in half of a revolution?

Θ = __π radians__

Half a revolution means half a circle. If a full circle is 2π, then a half circle must mean π.



How many degrees are in a full revolution?

Θ = __360 degrees__

∴ __2π__ radians = __360__ degrees

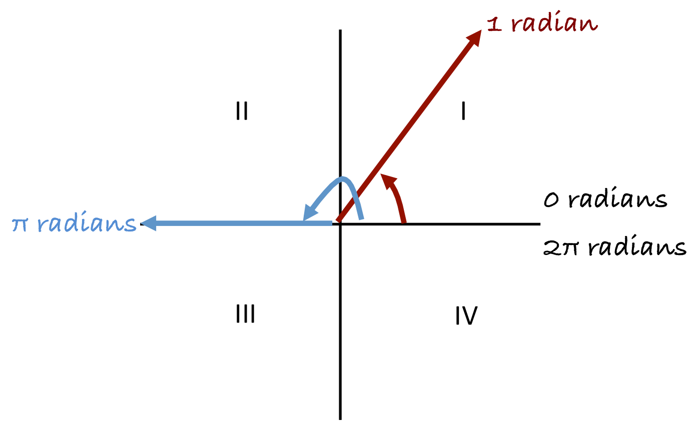
How many degrees are in half of a revolution?

Θ = __180 degrees__

∴ __π__ radians = __180__ degrees

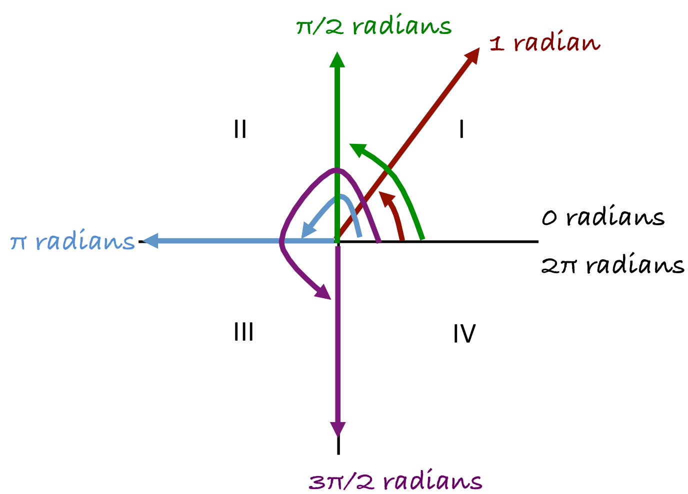
Lesson: Estimating angles (in standard position) with radian measures.

Begin by labeling what you have already discovered on the coordinate plane: the positive x-axis is 0 radians and 2π radians, the negative x-axis is π radians, and approximately 60º is 1 radian.



Next, reason that the positive y-axis must be $\frac{\pi}{2}$ radians because it is halfway between 0 and π.

Extend that to reason that the negative y-axis must be $\frac{3\pi}{2}$ radians. Label those on the coordinate plane.

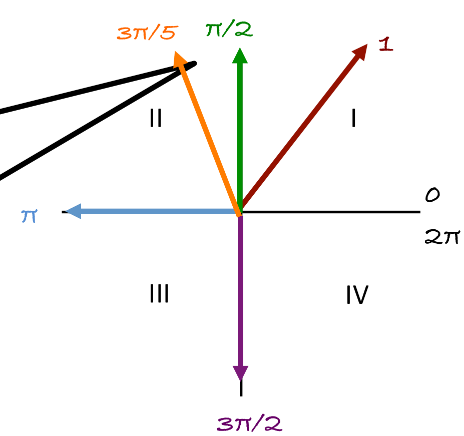


Use these benchmark angle measures to determine in which quadrant each of the given angle measures would lie. Do a think aloud with the first angle measure, $\frac{3\pi}{5}$ radians:

I know that $\frac{3\pi}{5}$ radians is less than π radians (because $\pi = \frac{5\pi}{5}$) but slightly greater than $\frac{\pi}{2}$ (because $\frac{\pi}{2} = \frac{2.5\pi}{5}$).

I will put the $\frac{3\pi}{5}$ ray a little bit after $\frac{\pi}{2}$.

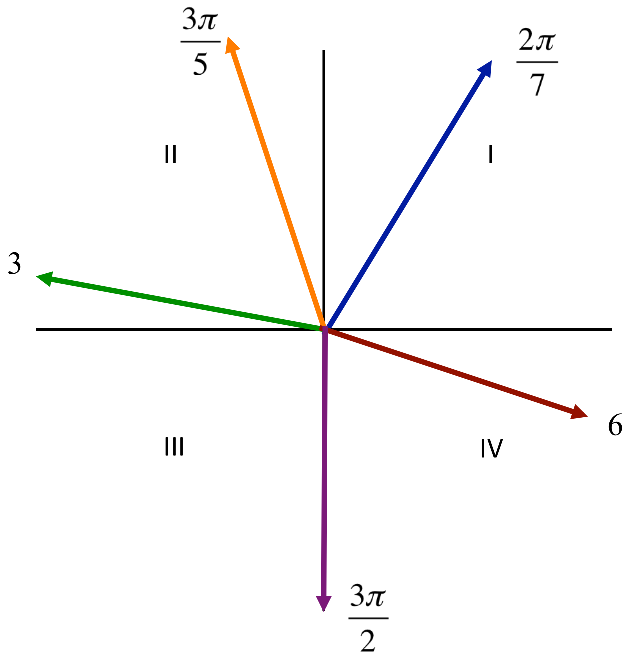
Therefore, $\frac{3\pi}{5}$ is in Quadrant II.



Lesson Solutions:

If each angle begins on the positive x-axis and rotates counter-clockwise, where would its second ray lie? May be in a quadrant or on an axis.

Sketch an estimation of each ray.



A) $\frac{3\pi}{5}$ radians **Quadrant II**

B) $\frac{2\pi}{7}$ radians **Quadrant I**

C) 3 radians **Quadrant II**

D) 6 radians **Quadrant IV**

E) $\frac{3\pi}{2}$ radians **negative y-axis**

Assessment: Exit ticket. Student copies are available on page 9.

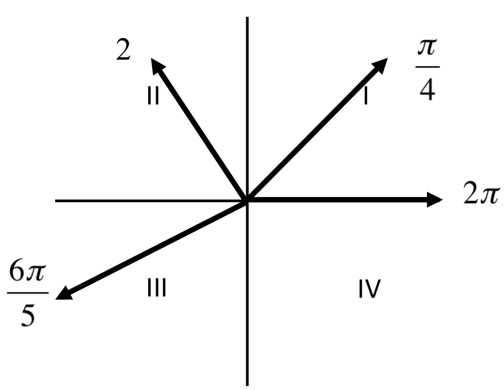
Assessment Solutions:

1) Explain (eg. draw, define) what a radian is:

Solutions here will vary.

2) If each angle begins on the positive x-axis and rotates counter-clockwise, where would its second ray lie? May be in a quadrant or on an axis.

Sketch an estimation of each ray.



A) 2π radians **positive x-axis**

B) $\frac{\pi}{4}$ radians **Quadrant I**

C) 2 radians **Quadrant II**

D) $\frac{6\pi}{5}$ radians **Quadrant III**

SWBAT Define a radian and determine its quadrant. Name____________________ Date ______ Period ___

Measure the diameter of your circular object: _______ cm ______ cm ______ cm

Use the group's agreed upon measure of the diameter to find the radius. RADIUS (r) = ________ cm

Cut the strip of paper to the length of the circumference of your object.
On that strip of paper, mark all the radius lengths that fit in the circumference.

Record other student's objects, radii, and how many radius lengths fit in their circumference:

Object	Radius length	# of times radius fits into circumference

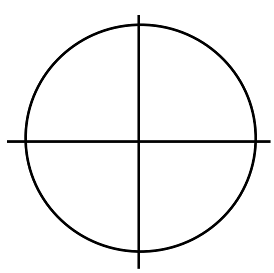
About how many radius lengths fit in your circumference measurement? _______________

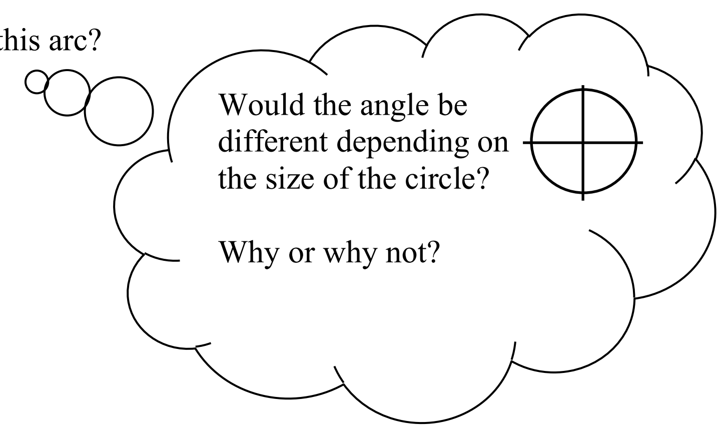
Does this number remind you of anything you learned in Geometry? ____________

If so, what? ____

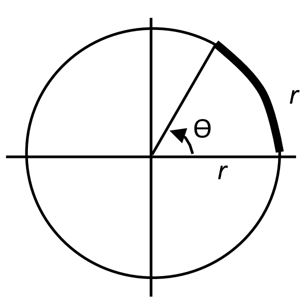
Draw an arc on the circle below that is one radius long.

What would the angle measure be for an angle to subtend this arc?





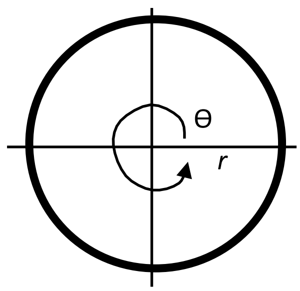
An angle that measures___________________ subtends an arc that is ________________ long.



Θ = ____________

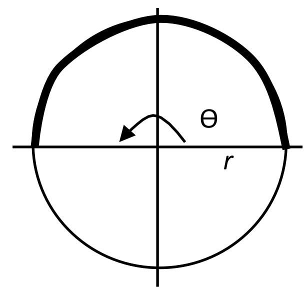
How many radians are in a full revolution?

Θ = ____________



How many radians are in half of a revolution?

Θ = _____________



How many degrees are in a full revolution?

Θ = ____________

How many degrees are in half of a revolution?

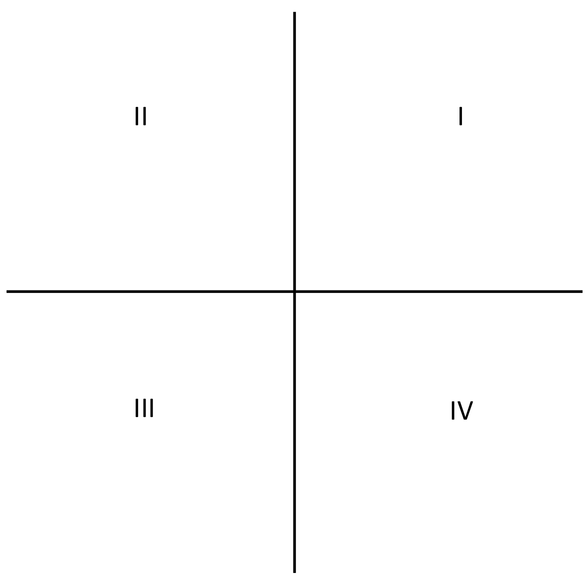
Θ = ____________

∴ _______ radians = ________ degrees

∴ _______ radians = ________ degrees

If each angle begins on the positive x-axis and rotates counter-clockwise, where would its second ray lie? May be in a quadrant or on an axis.

Sketch an estimation of each ray.



A) $\frac{3\pi}{5}$ radians ________________

B) $\frac{2\pi}{7}$ radians ________________

C) 3 radians ________________

D) 6 radians ________________

E) $\frac{3\pi}{2}$ radians ________________

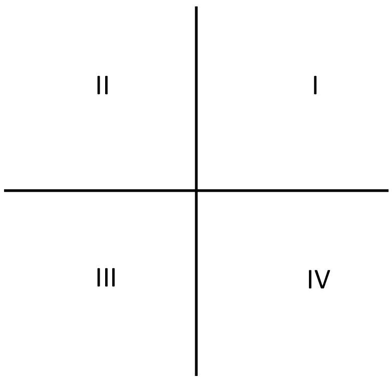
EXIT TICKET

Name: ____________________ Date _______ Period ___

1) Explain (eg. draw, define) what a radian is:

2) If each angle begins on the positive x-axis and rotates counter-clockwise, where would its second ray lie? May be in a quadrant or on an axis.

Sketch an estimation of each ray.



A) 2π radians ________________

B) $\frac{\pi}{4}$ radians ________________

C) 2 radians ________________

D) $\frac{6\pi}{5}$ radians ________________

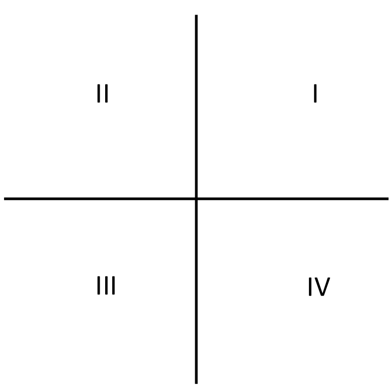
EXIT TICKET

Name: ____________________ Date _______ Period ___

1) Explain (eg. draw, define) what a radian is:

2) If each angle begins on the positive x-axis and rotates counter-clockwise, where would its second ray lie? May be in a quadrant or on an axis.

Sketch an estimation of each ray.



A) 2π radians ________________

B) $\frac{\pi}{4}$ radians ________________

C) 2 radians ________________

D) $\frac{6\pi}{5}$ radians ________________